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LOW-FREQUENCY ROOM RESPONSES:

Part 1 – Background and qualitative considerations

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Summary

The problems inherent in the reproduction of high-quality sound in relatively small rooms are discussed. Methods of predicting the inevitably uneven low-frequency response from source to listener or microphone are also discussed. It is concluded that, even with perfect prediction methods, there would still be some compromises which could only be resolved by making subjective judgements. The use of additional, low-frequency loudspeakers in control rooms permits an extra degree of freedom in the positioning of the loudspeakers to obtain the maximum degree of uniformity in response without compromising the stereophonic image localisation.

Index terms: *Acoustics; acoustic treatment*

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1. INTRODUCTION

The study of sound energy propagation within rooms has a long history[1, 2]. The beginning of the modern scientific approach lies at the end of the nineteenth century with the theoretical work of Lord Rayleigh[3] and the practical methods of Sabine[4]. Because of the very great complexity of the analytic solutions, much of the applicable work has centred on the treatment of the statistics of a hypothetical, random distribution of sound energy, sometimes including the non-diffuse, direct sound as an additional term. It has long been recognised that this simplification is invalid when the size of the enclosure is not very large compared with the wavelength. Many of the earliest workers recognised this and, despite the lack of modern computer processing, did much to treat the problems in a numerical way[5].

The principle difficulty, for the BBC, is the unevenness of the low-frequency response in most studios and control rooms. This is not so much a problem of the unevenness itself, because the majority of listeners' environments also respond in a different uneven way, but one of consistency. Given a 'standard' monitoring environment, a consistent product could be created by making some fixed allowances for the differences between the response in that environment and the 'ideal' sound quality (if that itself could be quantified). In the absence of both a standard room and the idealised target, all quality judgements are made more or less independently by many individuals. Overall control is by discussion and assessment of completed programme material in many different environments.

The work reported in this document arose from an increasing awareness amongst operation staff that these room-to-room differences were very large and were capable of having significant effects on the broadcast sound quality. It offers nothing new or revolutionary in the theoretical treatment of sound in small spaces; in many ways it is a review of existing knowledge. It does offer a reasonably compact precis of the acoustic considerations and may serve to promote further discussion.

2. THE PROBLEMS

2.1 What is sound and how is it propagated and perceived?

Sound energy consists of fluctuations of some physical attribute of a compressible medium; for example, the position of an elementary point in a solid or of the local instantaneous pressure in a fluid. It propagates in the form of waves, by virtue of the mass and elasticity of the medium. Such waves, falling within a certain range of frequencies, produce the sensation of hearing. For human beings, the range of frequencies which are audible is usually taken to be about 30 Hz to 20,000 Hz, although these limits depend significantly on the sound level and the age and otological history of the individual.

Because air is a fluid medium (that is, it cannot support shear stress) airborne sound energy can only travel as compression waves, with the instantaneous pressure at any point being alternately above and below the static atmospheric pressure. These waves propagate at a speed of about 340 m per second in straight lines with very little inherent energy loss with distance (at least at medium and low frequencies). Close to a small source, the wavefront, that is the line joining points of equal pressure, is spherical and the sound power emitted by the source is spread over a progressively increasing area, leading to a fourfold reduction in intensity for each doubling of distance. It is this 'spreading' effect which is responsible for most of the apparent reduction in sound levels with distance in the vicinity of small sources.

2.2 Sound energy in rooms

In practice, and particularly inside rooms, the sound energy emitted by a source does not travel for very long before its path is obstructed by some solid object. Even in a large room, such as a television studio or auditorium, this will occur within about 30 ms (10 m). In a small room, this first interaction may well be within one or two milliseconds. What happens then is invariably very complicated. Because of the wave properties of the sound energy, the effect of an obstacle depends on its size relative to the wavelength. Just as in the analogous situation with light, very large objects (relative to the wavelength) will completely obstruct the wave and possibly reflect some or all of it in one or more different directions, the remainder being absorbed. Smaller objects will distort the wavefront, this diffraction causing some of the sound energy to be diverted into different directions. Very small objects will not affect the wave at all.

Unfortunately for the acoustician, the range of wavelengths of normal sound lies between 20 mm and 10 m. This nicely encompasses the sizes of features often contained within rooms, including the size of the room itself! Thus, over the whole sound frequency spectrum, many different ratios of wavelength to object dimension are encountered. This means that the room design must deal with the entire range of effects from complete reflection or obstruction to insignificance, through all scales of diffraction.

Additional complications are introduced by the acoustic properties of the various surfaces within the room, with which the sound field might interact. Some surfaces are intrinsically hard and acoustically reflecting but others may absorb some of the sound energy or modify the shape of the wavefront by virtue of a reactive component of acoustic impedance. Furthermore, in typical sizes of rooms, the sound waves might travel many times across the room interacting with all the objects and surfaces encountered, before becoming insignificant. Typically, the sound might travel a distance of 20 or 30 times the largest dimension of the room in the time taken to fall in level by 60 dB, which is approximately the range taken for a reasonably loud sound to decay into inaudibility.

Because of this very complicated behaviour over a long period of time, a complete analytic approach to the description of the sound field, even in a simple real room, is not feasible. The problem may, however, be divided into three distinct frequency ranges — low, middle and high. The basis of the division is essentially that of the relationship between wavelength and object dimensions. At low frequencies the wavelengths are so long that only the principle dimensions of the room are relevant. At high frequencies many objects are large compared with the wavelength and cause both specular reflection and selective absorption of some parts of the wavefronts; both of these factors lead to discrete reflections in the short term and to highly diffuse* spaces in the longer term. Between these two frequency bands is a region of medium frequencies where some or all of these factors are operative.

This Report is concerned with the 'low' frequencies. It also assumes throughout that the sound energy detector is sensitive to sound pressure, as are most single-element measuring systems and, at low frequencies, animal hearing systems. An exactly equivalent (dual) set of descriptions could be used for velocity-sensitive detectors, but these are of less practical utility. Descriptions in terms of sound intensity would be different, but there is no way to detect intensity directly.

Throughout this Report, the phrase 'rectangular room' will be used to denote a three-dimensional space in which all of the boundary internal angles are right angles. Strictly speaking, the description should be 'rectangular parallelepiped' or 'right rectangular prism'.

* Diffuse spaces are those in which the sound energy is evenly dispersed throughout the volume and, at any instant, is travelling in a highly randomised pattern of directions.

3. LOW FREQUENCY ROOM MODES (EIGENTONES)

3.1 Solutions to the wave equation

Analytically, any acoustic enclosure, such as a room, may be considered as a three-dimensional space bounded by surfaces of (generally) complex impedance. The acoustic wave-equation for such a bounded space may be solved to give solutions in the form of eigentones, with characteristic frequencies, damping factors and spatial pressure distributions. In general, this method of solution is not feasible for anything but the most trivial idealised cases[6]. The eigentones and their associated spatial distributions (eigenmodes, or simply 'modes') may be considered from a less analytical viewpoint. If one considers a sound wave approaching and being reflected from a room surface in the direction of the normal to that surface, then the incident and reflected waves will be coincident (but travelling in opposite directions). In a rectangular room, such a reflected wave will eventually be reflected again from the opposing surface, to complete the circuit. If the wavelength happens to be simply related to the room dimension, then the reflections will be phase synchronous. Two such waves travelling in opposite directions will establish a standing wave pattern, or mode, in which the local sound pressure variations are consistently higher in some places than in others. This will occur at those frequencies for which the room dimension is an integer multiple of one-half wavelength. For example, in a room of 6 m length, this will occur at a lowest frequency of about 28 Hz and then at all multiples thereof. Similar patterns of standing waves will also be present for the other two pairs of surfaces. Furthermore, this triple, infinite subset of 'axial' modes is only one of three types of mode, the other two being 'tangential', involving reflections from four surfaces in turn, and 'oblique', involving reflections from all six surfaces.

In general, a mode will exist at a frequency given by the Rayleigh Equation[3] :

$$f = \frac{1}{2\pi c} \sqrt{\left(\frac{n_L}{L}\right)^2 + \left(\frac{n_W}{W}\right)^2 + \left(\frac{n_H}{H}\right)^2} \qquad (1)$$

where n_L, n_W, and n_H are the set of positive integers (including 0),

L, W and H are the room dimensions,

and c is the sound wave velocity ($\approx$ 340 m/s).

These modes have the attributes of resonant systems by virtue of energy transfer and storage mechanisms; that is, they have characteristic natural resonance frequencies, bandwidths dependent on their individual loss (damping) factors and amplification factors, also dependent on the damping. As in any other resonant system, the energy storage takes the form of cyclical interchange between kinetic and potential energies (or their equivalents). A way of visualising at least a fundamental axial mode is to consider the volume of the air within a room being divided into two types, the middle part acting as a mass, oscillating from end to end and being resisted by the stiffness of the end parts acting as springs. In the middle, no pressure is generated and energy can only exist as kinetic. Conversely, at the walls, no velocity components may exist and the energy of the field is entirely potential.

3.2 Modal densities and overlapping

In a small room such as a control room, listening room or small studio, typical values for the fundamental frequencies of the axial modes are 28, 35 and 45 Hz. Thus, in such rooms, the lowest part of the acoustically important frequency range, up to about 120 Hz, is characterised by a relatively small number of discrete room modes. At or near to each of these frequencies, sound energy will be amplified by the resonance effect to a degree dependent on the damping factor. At all frequencies above some critical lower limit, there are very many modes present in a typical room and this amplification is the norm for listening to sound inside rooms. At frequencies between the modes this amplification will not occur. This will be perceived as a relative attenuation. Thus, the response will be irregular. In practice, nothing can be done to eliminate the fundamental modal structure of a room, but if the proportions of the room are chosen carefully the coincidence of several modes can be avoided. Rooms whose dimensions are simply related are particularly subject to coincident modes. Rooms which are square or, worse still, half of a cube, are poor indeed.

If the modal density is fairly high it may be approximated by[7]:

$$N \approx \frac{4\pi f^3 V}{3c^3} + \frac{\pi f^2 S}{4c^2} + \frac{fL'}{8c} \qquad (2)$$

where N is the number of modes below the frequency f,

$V = LWH$ is the volume,

$S = 2(LW+WH+HL)$ is the total surface area

and

$L' = 4(L+W+H)$ is the sum of the edge lengths.

Equation 2 shows that the number of modes in any frequency interval increases very rapidly with frequency. (For the majority of reasonably-shaped rooms, the cubic term actually dominates the expression at all frequencies where the expression itself may be considered a reasonable approximation, that is for room volumes greater than about $2 \times 10^8 / f^3$)

At some frequency, the average modal density will be so high that the mode resonances will overlap significantly. This frequency marks the upper limit of the low frequency region of a room. One commonly accepted criterion for overlap is that the spacing between modes is equal to half the bandwidth, B, given by:

$$B = \frac{2.2}{T_{60}} \qquad (3)$$

where T_{60} is the reverberation time.

By differentiating Equation 2 an expression for the modal density can be obtained (after Ref. 7):

$$N \approx \left(\frac{4\pi f^2 V}{c^3} + \frac{\pi f A}{2c^2} + \frac{L}{8c} \right) \cdot \delta f \qquad (4)$$

where N is the approximate number of modes with eigenfrequencies in the bandwidth δf, and the other terms are as before.

By equating the expression from Equation 4 with that from Equation 2, an expression can be obtained for this limiting frequency, f_1, in any room (see Appendix):

$$f_1 \approx \sqrt{\left(\frac{Ac}{16V}\right)^2 - \left(\frac{L}{8c} - T_{60}\right) \cdot \frac{c^3}{4\pi V}} - \frac{Ac}{16V} \qquad (5)$$

For a typical small room (6 m $\times$ 5 m $\times$ 3 m) this gives a limiting frequency of 75 Hz, which is much lower than experience suggests, and is indeed lower than the limit for reasonable validity of Equations 2 and 4. A more reasonable statistical criterion might be five modes per bandwidth, the expression would then be:

$$f_2 \approx \sqrt{\left(\frac{Ac}{16V}\right)^2 - \left(\frac{L}{8c} - 2.T_{60}\right) \cdot \frac{c^3}{4\pi V}} - \frac{Ac}{16V} \qquad (6)$$

and would give a limiting frequency of about 120 Hz.

3.3 Effects of listener position

Because of the 'stationary' nature of low-frequency room modes, their effect is strongly dependent on the position of the listener. For any mode there will exist pressure nodes at which there can be no sound pressure at all (for an individual mode and for the theoretical case of zero damping factor), so that, however great the magnitude of the excitation, no sound pressure would be perceptible at such places. In practice — with many modes, finite damping values and frequencies imperfectly matched to the eigenfrequencies — the response at any point will be the vector summation, for all modes, of functions of position relative to the modal distribution and of frequency relative to the eigenfrequency.

3.4 Effects on the sound sources

The irregularity of the frequency response, as a result of the listener's location within the complex arrangement of standing waves, is only one half of the problem — the other is the location of the sound source within the same pattern. The listener at least is usually free to move in order to arrive at some kind of subjective impression of the average response. The source, especially if it is a loudspeaker in a control room, will usually be immovable. It is subject to an additional effect of the modal pattern — that of modification of its actual sound power output by the sound field[8, 9]. Many sound sources, including most loudspeakers, are actually generators of volume velocity rather than sound power. The delivered acoustic power arises as a result of the cross-product of volume velocity and dynamic pressure. If the source is located at or near a pressure node, then the acoustic pressure load (radiation impedance) is low for one direction of radiation and the output power coupled to that mode is correspondingly low. This results in reduced actual sound output power, which is perceived throughout the room as a dip in the frequency response. This effect is in addition to the irregularity of the modal coupling as a result of the listener's position.

3.5 Combination of source and receiver positions

In a real listening or recording situation, the source-to-receiver coupling factor is the product of the respective effects of room modes on the source and receiver.

To illustrate the magnitude of these effects, Fig. 1 **(a)** shows a typical calculated response for a particular room and positions of source and receiver taken from Ref. 10. Fig. 2 shows some of the 200 individual modal responses which contributed to that overall response. This may be compared with the actual measured response for the same conditions, shown in Fig. 1 **(b)**. This irregularity is currently one of the most serious acoustic design problems for sound control rooms. Within the BBC, it was identified at least as early as 1936[11], but has been essentially accepted until recently. It has become the source of significant complaints, much of the blame being incorrectly attributed to the loudspeakers.

It may be noted that many control rooms are about 6 m wide, with the stereophonic loudspeaker pair located carefully at the points of symmetry, about 1.5 m in from each wall. This is precisely at the pressure nodes of the second harmonic of the room width, which occurs at about 57 Hz. Many of these rooms show significant dips in their measured frequency response in the 63 Hz one-third octave band[10].

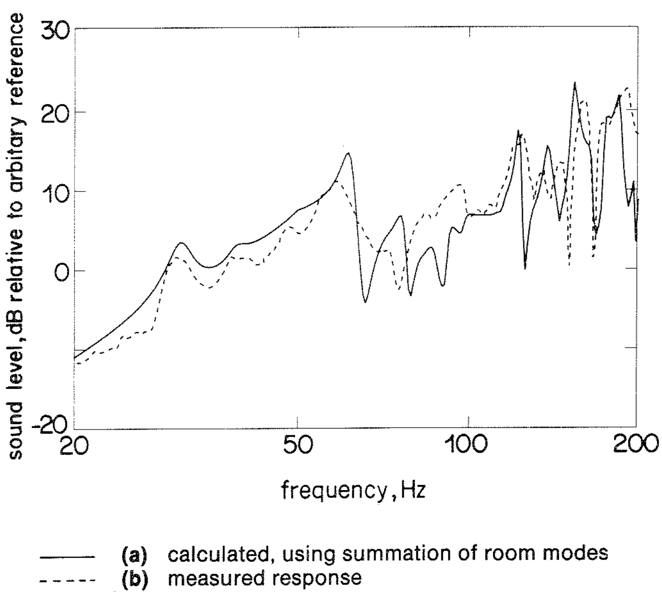

Fig. 1 - Low-frequency response, from Ref. 10.

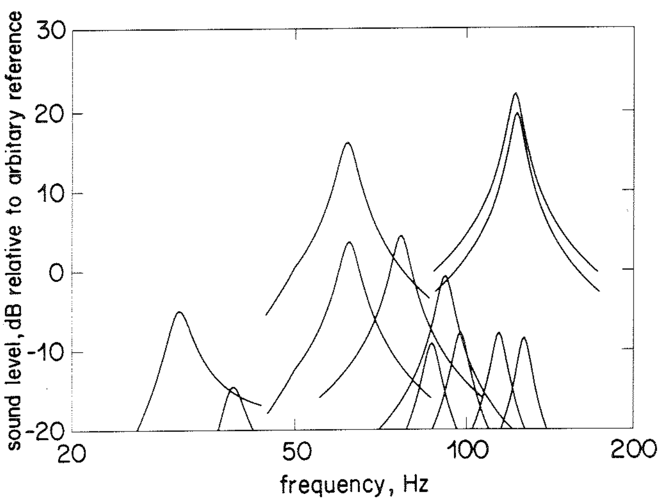

Fig. 2 - Sample selection of 12 modes, making up the response of Fig. 1 ***(a)*****.**

4. POTENTIAL REMEDIES

4.1 Increasing the modal damping factors

One method which may be employed to reduce the magnitude of these effects is to increase the damping factors of all of the modes by suitable disposition of acoustic absorption which is effective at these low frequencies[12, 13]. This is generally done to some extent as part of the routine overall design, in order to control the measured 'reverberation time' and thereby avoid excessively 'boomy' conditions. (In fact, at low frequencies in reasonably small rooms, the reverberation time strictly cannot be either defined or measured because it is a statistical property of completely diffuse spaces.) Much better control of the low frequency response can be obtained by consideration of the lowest modal frequencies individually and then selecting and placing acoustic treatment to deal with each one. By increasing the damping factor, the maximum amplitude of the resonance is reduced and the range of frequencies over which it acts is widened. The latter reduces both the effective mode spacing and lowest frequency at which the modes may be considered to be overlapping, that is the critical low-frequency limit. However, it is likely that a room so treated, to the extent of eliminating the subjective irregularities, if that were possible, would sound 'lifeless' and severely lacking in bass response.

4.2 Golden ratios

It is often suggested[14] that a pair of 'golden ratios' may be used to select the room proportions and, indeed, the use of such non-simple ratios is helpful, but the essential problem remains. Much work has been done[15 - 19], on the selection of such ratios. Using some criterion of modal spacing, such as the root-mean distance between adjacent modes for frequencies up to the critical limit, distribution maps showing the 'quality' may be generated for ranges of room proportions[15]. Fig. 3 shows such a distribution contour map for a rectangular room of 200 m^3, using a simple rms mode spacing criterion for all mode frequencies up to 120 Hz. The axes are length and width as multiples of the room height. For nearly-square rooms, ratios near to 1.40 and 1.19 times the room height are optimum (that is, giving minimum value of mean-squared mode spacing). For rooms in which the width and length are both about twice the height, proportions near 2.28 and 1.89 are optimum.

The general appearance and location of optima in the resulting contour maps are not very sensitive to the exact nature of the 'quality' criterion. Similar values for these optimum ratios are obtained for largest individual mode spacing (1.43 : 1.18 : 1 and 2.28 : 1.71 : 1) and mean fourth-power of mode spacing (1.43 : 1.18 : 1 and 2.30 : 1.75 : 1).

4.3 Non-rectangular rooms

It is also suggested that a possible solution is to make the room non-rectangular. Before discussing this, it may be useful to consider how it is that the surfaces of a room can cause low-frequency specular reflections when the wavelength is comparable with or even

Figures are 10 × mean square mode spacing, higher 'quality' = lower mean spacing.

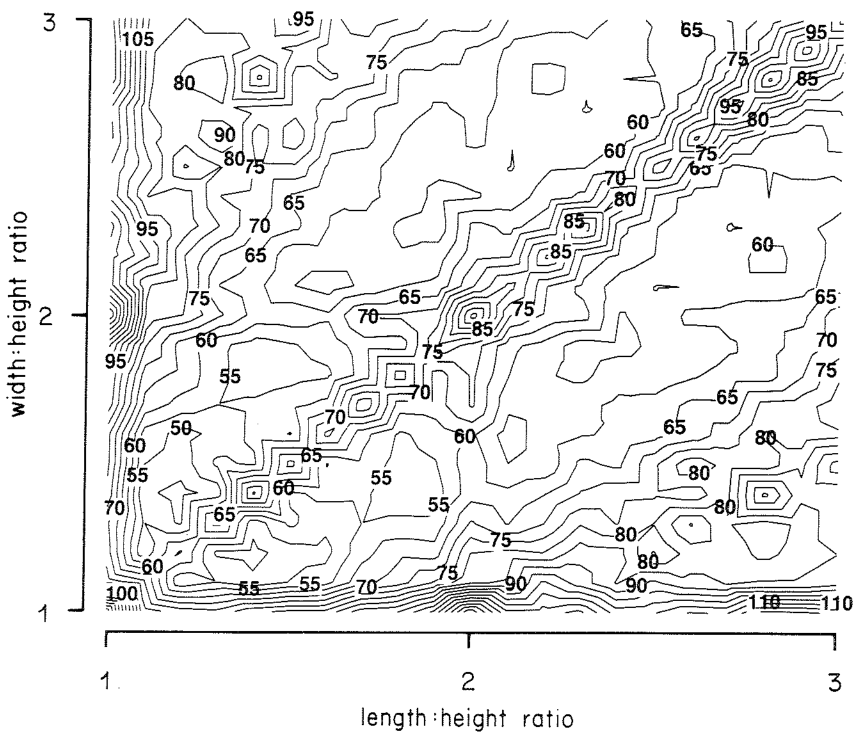

Fig. 3
Contour plot of room 'quality', for 200 m^3 room, using mean square mode spacing for frequencies up to 120 Hz.

greater than the extent of the surface. In an otherwise free space, such a reflection would be diffraction limited. Modes can only exist because the reflections are more or less specular. However, if one considers a single boundary surface of a rectangular room, the virtual images of that surface, formed by reflection in the adjoining surfaces, effectively extend the real surface to infinity. Thus, acoustically, a rectangular room consists of three sets of interlocking infinite planes. Of course in practice, any boundary absorption eventually truncates the planes, but the effect is to produce large reflectors in each principal direction. The effective reflection may, therefore, be specular, even though the physical surfaces are small.

If one then considers a deviation from parallel of one of the pairs of surfaces, for example a tapering of the width of the room towards one end, the images of that taper will alternate in angle, producing effective surfaces which have a 'ripple-like' structure. Unless the peak-to-peak magnitude of the ripple is a significant fraction of the wavelength, this will have no effect except to shift the effective reflector to the mean position; that is, the room width at the mid-point of the side walls. To have any other effect at, say, 60 Hz, the extent of the taper would have to be of the order of 1 - 2 m per side!

Many modern rooms are significantly non-rectangular, generally for reasons other than the low-frequency modal behaviour. In these cases, the mode structure cannot be derived by simple equations or by inspection (although approximate methods based on mean dimensions may give sufficiently accurate results for most purposes). Mathematical modelling techniques like Finite Element Analysis[20 - 23] can, in principle, give arbitrarily accurate answers in such cases. However, the modal behaviour is no less pronounced than in an equivalent rectangular room — just different!

4.4 Relocation of sound sources

Another factor often employed in control rooms, again usually for other reasons, is the incorporation of loudspeakers into the walls. Theoretically, all modes have pressure antinodes at their reflection points. Thus, a hard wall surface is a pressure antinode for one third of the axial modes, two thirds of the tangential modes and all of the oblique modes. Placing the sound source there ensures that it is better coupled to more of the room modes. Such considerations can help to reduce the response irregularities, at least for the coupling of the loudspeaker to the room. However, it is generally unrealistic to consider loudspeakers located in the extreme corners of the room. Also, such an approach does nothing whatever about the irregularities resulting from the listener's position with respect to the room modes. It may well be that an excess response at one frequency, as a result of the listener's (or microphone) position, requires a degree of partial cancellation by locating the loudspeaker away from the position giving most effective coupling.

One arrangement in which the loudspeakers can be located at other than the normal stereo listening positions, without detriment to the higher-frequency image localisation, is the use of 'sub-woofers'[24]. Because the problem is one of low-frequency response and there can be no image localising information at frequencies of the order of 100 Hz (consideration of time-frequency resolution limitations simply does not permit it), the extreme low-frequency part of the spectrum could be handled by loudspeakers specially located to achieve a more even response. This arrangement may also have side benefits in loudspeaker design and power-handling capabilities.

Much of the above discussion has been related to the reproduction of sound in a control or listening room. All of it is relevant to studios and live performance spaces — they also have sources and listeners (or microphones) subject to precisely the same considerations. However, many of these spaces are larger, some very much larger, so that the critical frequency limit may be very low indeed. In concert halls, the modal density at all reasonable frequencies is high and these low-frequency problems do not exist at all for the wanted sound. However, some cases of very low-frequency resonances causing problems have been recorded in other applications[25, 26, 27].

4.5 Electronic and electroacoustic equalisation

Many proposals for response correction by purely or partial electronic systems have been proposed[28]. Most of these attempt to address the problems over the entire frequency range. Until fairly recently, it has been (and in at least one case, still is) common practice to include equalisation in the reproduction chain in order that the measured frequency response, at a predefined listening location, can be measured and made to match some predetermined ideal. However, this is usually implemented using ⅓rd-octave bandwidth devices and bears little relationship either to the human perception of sound or the fine detail of the response irregularities.

For high frequencies, the room mode distribution is so dense and complex, leading to very complex frequency-domain irregularities, that no practicable system of correction based on the frequency response could be envisaged. Instead, most of the proposed systems invoke the duality of the frequency and time-domains to allow some corrections to be made in the time domain. In order to make the necessary filters realisable, this is generally limited to just the first few discrete reflections. Apart from the very serious, possibly insurmountable, problem of large changes in the responses for small changes in the source or listener positions, such systems are capable of correcting impulse responses for the first 50 - 100 ms at the cost of additional, pseudo-random artifacts in the longer-term. In practice, this does at least coincide with the usually-accepted view that the human perception of sound fields is dominated by phase and time-domain representation at higher frequencies. The study of high-frequency room design has been the subject of other work[29].

At low frequencies in monitoring rooms, there are valid reasons for introducing electronic correction in order to compensate for irregularities introduced as a result of the loudspeaker interaction with the discrete or relatively widely-spaced room modes. The effect of an isolated room mode on the effective output power of the loudspeaker is essentially indistinguishable from a real departure of the loudspeaker from a level response. Equalisation could remove such effects. However, it would not necessarily create a more uniform response at any particular point within a room, because the effects of the listener's position within the room mode structure may as likely as not be partially cancelling the adverse effects of the speaker position. For cases with fixed source and receiver positions, low-frequency equalisation could be applied, in principle to perfection, within the limitations of system power-handling capacity.

5. CALCULATION OF FREQUENCY RESPONSE FUNCTIONS

5.1 Solution of the wave equations

The entirety of the response in both time and frequency domains, for any combination of source and listener positions, can, in principle, be obtained from an analytical solution of the acoustic wave equation for the particular boundary conditions of the room. As already discussed above, the solution of this equation, for anything but the most trivial case, is difficult. Even if such a solution could be found for a given room shape and surface impedance, there would remain the problem of describing a large number of real materials in terms of their acoustic impedance. The latter is a problem common to all prediction methods; but, at least, in some other cases, the relationships between the surface impedance and the solution are easier to envisage from comparisons with measured results, and so enable first-order corrections to be made.

5.2 Finite-element methods

Numerical methods for solving otherwise intractable equations are common throughout science and engineering. They have the advantage of usually producing some form of solution to the hardest problems. If the approximation is adequate and well-behaved, then the answer may be arbitrarily close to the exact solution, at the cost of extended processing time. In acoustic Finite Element Analysis, the sound field is generally represented by a number of elemental regions, each one of which is so small that the expressions for continuity of volume velocity, pressure, mass, etc. may be adequately represented by low-order (usually first or second) polynomial equations. This requires that the elements are much smaller than a wavelength. A limit of about $\frac{1}{10}$th wavelength is generally regarded as the largest which can give meaningful results for a first-order representation and twice that for the second-order[30]. In any reasonable space this corresponds to a very large number of elements, about 200 per m^3 for an upper limit of 200 Hz. In a typical control room this would require about 20,000 elements, each one of which requires three-dimensional continuity equations for mass, volume velocity and pressure. For a modern computer processor, the solving of this set of equations would be relatively simple and quick. However, to obtain a frequency response for a given pair of source and receiver positions would require a new solution for every frequency. This, and the labour involved in specifying and entering the boundary conditions for the walls, and any substantial objects within the space, would represent a very significant amount of effort. This method also suffers from the requirement to know the acoustic properties of all of the individual materials in some detail.

An alternative to solving the set of equations for each frequency would be to derive the eigenmode parameters for the room. The frequency response could then be calculated by summation of the modes (as in Section 5.4 below).

5.3 Reflection summation

The net response for a particular listening position in a room is frequently described in the form of the summation of sound rays as though from images of the source, reflected in the space boundary surfaces. One branch of acoustic design actually seeks to formalise this method; known as 'ray tracing', it has significant support amongst the designers of rooms of all types. In general, it is more easily applied to high frequencies or large rooms (or both). In these cases the complications caused by diffraction are less severe. In the special case of low-frequency reflections in small rooms, the principle of large effective reflectors (generated, as described above, by the images of wall surfaces) may be invoked to permit this summation for low frequencies in relatively small, nearly-rectangular rooms. Some work has been done using this method, but omitting the effect of the room on the sound source*. Because the number of higher order reflections increased exponentially, it was found that their contribution to the total response was significant, despite the progressive attenuation caused by fairly high (but realistic) values of absorption coefficient. The number of reflections which had to be included was, therefore, large (up to 4000). This summation had to be carried out afresh for each frequency. For rectangular rooms, it was also a trivial matter to determine the location and 'visibility' of images — a simple expression was all that was required. In a non-rectangular room, the determination of these parameters is less obvious and would involve very much more arithmetic, most of it to no avail because low-frequency sound does not actually behave like that!

The potential accuracy of calculation also raises some doubts. In a large concert hall, where such methods have been proposed (and in some cases actually used), even to graze the edge of a 2 m diameter patch of material at a range of 50 m would require a single reflection to be predicted to within an accuracy of $\pm 1°$. It is quite beyond the bounds of credibility that the acoustic parameters of real materials could be known and the angle of reflection calculated to a degree of accuracy sufficient to make that a worthwhile exercise. Progression to higher-order reflections is to invite comparison with the theories of 'chaotic' systems. In an idealised world of infinite frequencies and perfect surfaces, such things are possible but their connection with real acoustics is, perhaps, tenuous.

5.4 Mode summation

The method of mode summation is one of the subjects of the companion Report[10]. In principle, for any one mode, the coupling of a source to a room and the room to the receiver are both functions of their positions relative to the (stationary) modal pressure distribution. In a rectangular room, these modal distributions are cosine functions of one, two or three dimensions (assuming an origin in the corner of the room), depending on the type of mode. The transfer function from source to receiver can be represented as the product of two such cosine functions. It is also a function of the frequency relative to the mode eigenfrequency and of the damping factor, Q. The total response at any frequency is the vector sum of all such modal couplings, plus a term representing the direct sound field.

* Unpublished work carried out by J.A. Fletcher, BBC Research Department.

For this calculation, the modal frequencies need to be found once only and their values stored. In a rectangular room, this is a trivial matter. The coupling coefficients at source and receiver positions are, likewise, simple cosine functions which can be calculated once and also stored. In a significantly non-rectangular room, the same principle can be applied; Finite Element Analysis, or any other suitable method, being used to derive and store the eigenfrequencies and their coupling coefficients at source and receiver positions.

Figs. 1 and 2 (on page 4), taken from Ref. 10, show an example of the results of such a calculation. The overall response shown in Fig. 1 **(a)** is the vector summation of about 200 modes like those shown in Fig. 2. For comparison, Fig. 1 **(b)** shows a measured response for the same conditions. In fact, the detailed responses are so dependent on positions, even at low frequencies, that the measurements themselves are not entirely repeatable. However, there are clear indications that the overall characteristics of the response have been predicted, even if some of the frequencies and amplitudes are in error. Even the major characteristics at frequencies near to 200 Hz show significant similarities. The material parameters required for these calculations are the overall surface average values for large parts of the room boundaries. For typical studio and control room installations at least, the real parts of the surface impedances can be obtained from the reverberation time. The reactive components are likely to shift the eigenfrequencies somewhat, in a manner that could, in principle, be calculated from measurements of actual modal frequencies.

5.5 Limitations of predictions

Whichever method of prediction is employed, it will be subject to uncertainty because of imprecision of the input data, arising from lack of knowledge of the material acoustic properties. No method of calculation can overcome this limitation. The effective properties of acoustic materials are difficult enough to obtain to any degree of accuracy. Data for the acoustic properties of other structural or decorative materials is virtually impossible to obtain. At low frequencies many structures exhibit panel or Helmholz resonances, with associated large changes in impedance (particularly in the reactive component), over small frequency ranges. For example, attempts to identify and measure individual modes and their distributions in rooms rapidly become difficult above the frequency at which the first oblique modes appear. Some attempts to do this in an acoustically-treated room of 76 m^3, using up to six loudspeaker sources in an attempt to excite modes selectively, appeared to show that the modal frequencies were within 2 - 6% of their expected values up to about 100 Hz (Table 1). The

Table 1: Measured room modes.

Mode order (HWL)	Frequencies, Hz	
	Calculated	Measured
001	31.0	31.3
010	38.7	38.0
011	49.5	47.1
100	55.5	55.6
002	62.0	57.8
020	77.3	76.6
102	83.2	81.4
021	83.3	82.2
112	91.7	91.7
003	93.0	93.2
120	95.2	94.4
022	99.1	99.5
013	100.7	102.7
103	108.3	107.7

work described in the companion Report to this one[10] (some of which was carried out in the same room) illustrates some of these differences and their effects.

Such a lack of accurate data prompts the question of how far it is worth pursuing any method of prediction. Refs. 10 and 31 demonstrate that the peaks in the frequency responses for isolated modes are neither where they are calculated to be, nor quite as they were measured to be when taken in isolation. In the work described in Ref. 31, extensive experiments in a closely-specified and idealised model room failed to reveal any consistent reasons for the frequency errors. If this so obviously occurs where the modes are isolated, then it is probable that the same effects are happening where the modes are overlapping. Even at relatively low frequencies, where there might be 10 - 12 modes making significant contributions to the overall response, uncertainty in their frequencies can cause large changes in the summation, particularly if two large contributions are in phase-opposition. At higher frequencies there will be a significant degree of statistical averaging, just as there is in real rooms, but it is unlikely that any more than a general indication will be obtained.

6. PROPOSED DESIGN PRINCIPLES FOR MINIMISATION OF IRREGULARITIES

In attempting to set design principles, the first task must be to identify the target. It may be too simplistic to say that the response must be 'flat'. The majority of listener's environments probably exhibit a fairly pronounced bass rise in their reverberation time[32, 33, 34]. Against that, the majority of domestic listening is carried out with equipment having a significantly truncated bass response. Whether the monitoring/mixing environment should reflect and go some way towards compensating for either of these facts is a subject for the producers of the programmes. For the present purpose, it will be assumed that the overall system response at the studio manager's (sound supervisor's) position, including the effects of the loudspeakers and the room, should ideally be uniform down to some cut-off frequency. Below that frequency, which will be in the region of 40 to 100 Hz, depending on the programme material type, the loudspeaker response may fall off and the room effects will inevitably become irregular.

Many measurements have been carried out in working control rooms following complaints about response non-uniformity at low frequencies[10]. These have almost always shown very serious irregularities in the effective frequency response from loudspeaker to listener's position. In most cases, the very low frequency irregularities (up to about 100 Hz) are clearly associated with the geometry and equipment layout of the room. In one example, there was a pronounced emphasis at frequencies around 63 Hz. This exaggerated response was consistent across the whole width of the sound mixing desk because the mode concerned was an axial third harmonic of the room length, with the loudspeakers located at one pressure antinode and the desk at the next. The combined effect of these two factors amounted to about +10 dB. The reason for the uniformity across the width of the mixing desk was that the only other mode near that frequency, the second harmonic of the room width, was not excited because the loudspeakers were at the quarter-wavelength positions in the room width. In such easily identified cases, the 'simple' remedy is to relocate the loudspeakers to produce a more uniform response characteristic. It is a debatable point whether the presence of the second harmonic of the width would have improved or worsened this example. At such frequencies in the usual size of control room it is always going to be necessary to balance one defect against another.

At higher frequencies (between about 100 and 160 Hz) the measured responses are also somewhat irregular because, statistically, the density of modes is still small. However, the exact detail of the mode summation is, so far, unpredictable and greatly influenced by some large items of equipment and the detailed behaviour of the wall treatment. Towards the upper end of this frequency range the modal density is large enough for the room dimension ratios to be insignificant — many modes will coincide whatever the room proportions. Thus, there seems little that could be done in the design stage to prepare for these potential problems. Even at the acceptance stage there is very little that could be adjusted in a causal sense to correct defects.

The geometry, size and equipment layouts of most control room designs are usually predetermined by factors other than the acoustic performance. The pressure of cost usually precludes the provision of rooms of adequate size for the proper reproduction of low frequencies. One of the first acoustic design criteria should be the recognition that below some predictable frequency, f_2 (Equation 6), the response is bound to become somewhat irregular, and that below a lower frequency, f_1 (Equation 5), the response at any frequency will be dominated by only one or two modes (or even none at all) and is bound to be very irregular for some combinations of source and listener positions.

For frequencies up to f_1, the mode frequencies and pressure distributions should be individually calculated and the room proportions adjusted to minimise the mode overlap. Care should be taken to minimise large departures from uniform coupling from source to listener by adjustment of the room proportions and equipment and listener positions. If the general equipment layout is predetermined by other factors, as is usually the case, there will be very little scope for such changes, because the required magnitudes of change are quite large — at least ⅛th wavelength. One degree of freedom which sometimes exists and may be quite helpful, is the choice of orientation of the installation within the basic shell. The use of highly effective acoustic treatment will assist in maintaining low Q-factors and, hence, in avoiding large irregularities. This treatment should, preferably, be selectively adjusted to match the individual modal frequencies and located at the pressure antinodes for each mode. This use of effective low-frequency acoustic treatment runs directly against the current trend (driven by cost considerations) to minimise the use of such material and accept the consequential bass-rise in the reverberation time.

For frequencies between f_1 and f_2, the room proportions may be chosen to equalise the mean mode spacing. However, this may not make very much difference, provided that obviously deficient proportions such as half-cubes are avoided. Fig. 4 shows the calculated responses for two rooms of 100 m^3 volume. The first of these, **(a)**, has proportions close to the optimum for a room about twice as long and wide as its height, 2.28 : 1.89 : 1, as determined in Section 4.2. The second, **(b)**, is a room with what would generally be thought to be poor proportions, 7 : 5 : 3*. Although there is some evidence of greater uniformity in the former, it would be hard to argue that the difference was dramatic. The worst case which could be taken for such a room would be a half-cube, 2 : 2 : 1. The same comparison for such a room is shown in Fig. 5. In that case, the discrepancy in levels between 65 - 80 Hz and 90 - 120 Hz is fairly pronounced. The method of response calculation is described in Ref. 10. Although the method does lead to results which are not identical to measured ones, the comparisons should be valid because the method is common to all cases.

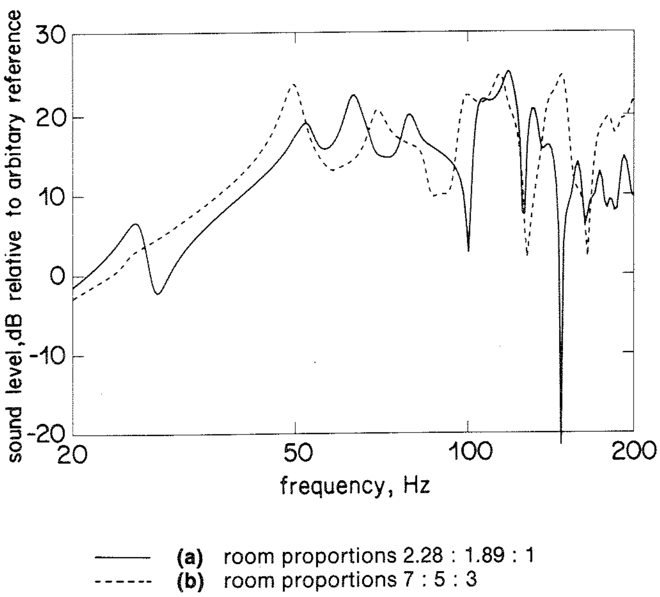

Fig. 4 - Calculated frequency response for two rooms of 100 m^3.

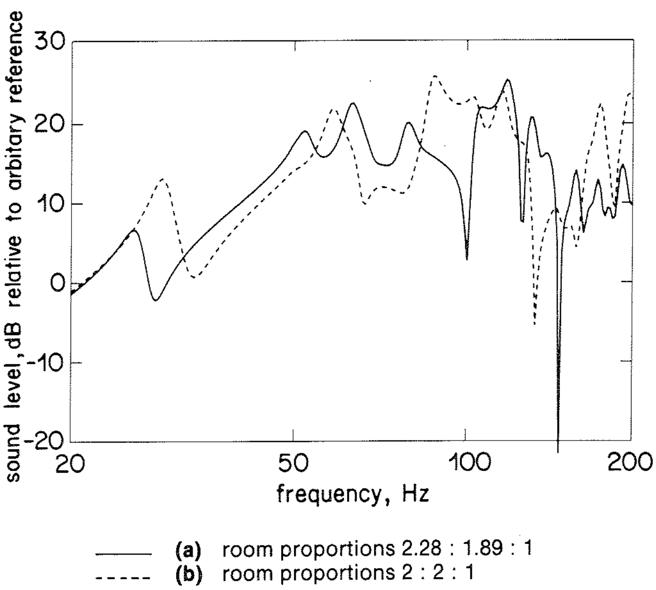

Fig. 5 - Calculated frequency response for two rooms of 100 m^3.

* Because of the conventional wisdom which declares that room proportions should not be in simple, small-valued integer ratios.

7. SUB-WOOFER LOCATION

One degree of freedom which is available for retrospective correction of low-frequency room response faults is the provision of additional low-frequency loudspeakers. Although colloquially known as 'sub-woofers', the intention in these cases is not to extend the low-frequency response below the normal limits. (However, in correcting the 'normal' low-frequency response, it could be argued that the effect is to extend the *usable* low-frequency response.) The main purpose is to permit relocation of the low-frequency sources to positions which result in a more uniform response, without sacrificing the higher-frequency, stereophonic performance.

Using a filter specially developed for the purpose, with crossover frequencies switchable between 80, 100 and 120 Hz, several experimental installations have been tested in areas subject to complaints from the operational staff. Most of these cases were found not to be improved subjectively by the addition of the sub-woofers, despite measured improvements in ⅓rd octave frequency response regularity. It can only be supposed that the substance of the complaints in these cases was not directly related to the objective irregularity of the response in the low-frequency region (up to 120 Hz). One case, however, (at the time of writing) was so improved that the operational staff were reluctant for the experimental system to be transferred to another location before they had constructed a replacement filter.

At the present state of knowledge, it is not possible to predict the optimum sub-woofer location in any but the simplest cases. It is usually necessary to resolve conflicting balances between several different frequency ranges, in spaces where potential additional loudpseaker locations are very few. In practice, a reasonable location can usually be found, often in the 'free' space behind the mixing desk, by trial and error, using a real-time, third-octave spectrum analyser for continuous display of the response to a single listening position. Once a position has been found which produces a more uniform response, it is a simple matter to adjust the relative gain of the sub-woofer channel for optimum balance. This control must then be 'locked' to prevent 'unofficial' adjustments.

No attempts have yet been made at improving the response at more than one listening position at the same time.

8. CONCLUSIONS

A discussion has been presented of the acoustic parameters governing the behaviour of low-frequency sound energy inside relatively small enclosures. The limitations imposed by these physical restraints on the achievable subjective sound reproduction quality have also been discussed. Much of this material exists in other works. It has been brought together, in this comparatively compact and descriptive form, to present ideas in relation to the special problems facing acoustic designers of high-quality sound studios and sound monitoring rooms; for economic reasons, these areas have to be relatively small. It also forms the descriptive background for a companion Report presenting the detailed results of calculations and measurements in real environments.

Some methods of predicting objective frequency responses have been outlined and their advantages and disadvantages presented. Most of these prediction methods require detailed information about material properties and behaviour which is unobtainable. Some are analytically intractable or computationally extravagant. Others bear little resemblance to real acoustics. One method, that of mode summation, is practicable, at least for rooms which can be approximated acoustically to the simple rectangular form, and gives results which resemble the measured response.

Some guidance on the design of small rooms to optimise the low-frequency response has also been given. However, it has to be said that the problem is, in general, insoluble. Even if perfect predictions could be made, the final result will inevitably contain large low-frequency irregularities. The optimisation of such responses, for different positions within the room, for the relative importance of different parts of the low-frequency spectrum and for different listeners' preferences, is a matter for subjective assessment rather than for any theoretical pedagogy.

One additional degree of freedom can be derived from the use of separate, low-frequency loudspeakers. This permits the optimisation of the low-frequency response without detriment to the higher-frequency, stereophonic imaging.

It has to be accepted that, in rooms, there will be a low-frequency limit below which the perceived frequency response will be irregular. For practical sizes of studios and control rooms, this frequency limit will be within the range normally considered to be part of the broadcast spectrum.

A fairly large reference section has been included for further detailed studies, but the book by Morse[5] leaves little of the theoretical background untreated.

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APPENDIX

Calculation of the Upper Limit of the Low-frequency Region

Equation 4 gives the approximate number of eigenfrequencies in any band of width δf centred on f Hz:

$$N \approx \left(\frac{4\pi f^2 V}{c^3} + \frac{\pi f A}{2c^2} + \frac{L}{8c} \right) \cdot \delta f$$

If the criterion for overlap is that the mean spacing should be half the bandwidth given by Equation 3, then $N = 1$ for $\delta f = 1.1/T_{60}$. Rearranging and solving for f gives a limiting frequency, below which the modes must be considered to be isolated, of:

$$f_1 \approx \sqrt{\left(\frac{Ac}{16V}\right)^2 - \left(\frac{L}{8c} - T_{60}\right) \cdot \frac{c^3}{4\pi V}} - \frac{Ac}{16V}$$

For a room of 6 × 5 × 3 m and a reverberation time of 0.3 s, this gives a value of 74.6 Hz.

At this limit, any mode bandwidth might contain one or two mode centre frequencies. This is clearly a sparse state, with little pretension to an idealised, statistical sound field. Another limiting frequency might be determined, for which the average modal density is sufficiently large that any modal bandwidth will encompass several eigenfrequencies and that the statistical variation between bands will be reasonably small. Such a limit is fairly arbitrary but five modes per bandwidth might be a reasonable figure. In that case, $N = 5$ for $\delta f = 2.2/T_{60}$. This may be approximated to $N/\delta f = 2.T_{60}$, and the expression for the limiting frequency becomes:

$$f_2 \approx \sqrt{\left(\frac{Ac}{16V}\right)^2 - \left(\frac{L}{8c} - 2.T_{60}\right) \cdot \frac{c^3}{4\pi V}} - \frac{Ac}{16V}$$

For the same room and reverberation time, this gives a limiting frequency of 117.4 Hz.

For reasonable rooms and reverberation times, characteristic of small studios and control rooms, these expressions can be simplified to:

$$f_2 \approx \sqrt{\frac{T_{60}\, c^3}{4\pi V}} - \frac{Ac}{16V}$$

and

$$f_2 \approx \sqrt{\frac{T_{60}\, c^3}{2\pi V}} - \frac{Ac}{16V}$$

respectively.

In 1954, M.R. Schroeder proposed a limiting frequency corresponding to a mean mode spacing of one-tenth of the mode bandwidth[35] :

$$f \approx 4000. \sqrt{\frac{T_{60}}{V}}$$

This is commonly called the Schroeder frequency, but is recognised as being conservative[36]. A proposal for three modes per mode bandwidth can be expressed as:

$$f \approx 2000. \sqrt{\frac{T_{60}}{V}}$$

These equations give frequencies of 231 Hz and 115.5 Hz respectively for the example room.